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Theoretical neutrino physics

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Abstract. I review the interpretation of the recent beautiful neutrino oscillation data, in particular the major breakthrough in the solar neutrino problem. Then I list the Seven Questions in neutrino physics and how we may address them in the future. In the end I try to put the neutrino masses and mixings in the context of the question of flavor, and conclude.

PACS. 14.60.Pq Neutrino mass and mixing

1 Introduction

The title of my talk was given by the organizers, which made me feel a little uneasy. I started to wonder what "Theoretical Neutrino Physics" was. It also reminded me of the claim Milind Diwan has been making in virtually every talk he gives:

> *"Neutrino physics is so simple. There are no hadronic corrections to worry about. We don't need theorists."*

In the end, I realized what the organizers meant. It is supposed to mean

Why You Need Theorists in Neutrino Physics

You will be the judge at the end of the talk.

It is clear that the neutrino physics is going through a stage of revolution. I'll begin my talk with a little historical perspective why theorists had high hopes in neutrino physics well before the recent big excitement. Then I'll move on to the interpretation of beautiful data, starting with the solar neutrino. The overall interpretation of the data depends on how we treat the claimed evidence for neutrino oscillation from the LSND experiment. I will discuss the interpretation without LSND first, and then with LSND. After that, I will discuss the issue of the nature of the neutrino masses. I will interleave seven big questions in neutrino physics together with the interpretation of the data. Finally, if I will have time left, I will discuss models of neutrino masses and mixings, or rather more generally models of flavor, and their consequences on future observables.[1]

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[1] In fact there was no time left for this discussion.

2 A little historical perspective

One way to characterize particle physics is as a field that tries to understand nature at its most fundamental level, namely at the shortest distance scales, or equivalently the highest energy scales, possible. There has been two ways to do this. One way to access physics at the highest energy scales possible is of course to build powerful particle accelerators. Another way is to look for rare effects from physics at high-energy scales that do not occur from physics at known energy scales, namely the Standard Model. Neutrino physics belongs to the second category.

Rare effects from physics beyond the Standard Model are parameterized by effective operators added to the Standard Model Lagrangian,

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 \cdots \quad (1)$$

The effects in $\mathcal{L}_5$ are suppressed by a single power of the high energy scale, $\mathcal{L}_6$ by two powers, etc. What terms there can be have been classified systematically by Weinberg, and there are many terms suppressed by two powers:

$$\mathcal{L}_6 \supset QQQL,\ \bar{L}\sigma^{\mu\nu}W_{\mu\nu}He,\ W^\mu_\nu W^\nu_\lambda B^\lambda_\mu, \\ (\overline{Q}_2\gamma^\mu Q_1)(\overline{s_R}\gamma_\mu d_R),\ (H^\dagger D_\mu H)(H^\dagger D^\mu H), \cdots \quad (2)$$

The examples here contribute to proton decay, $g-2$, the anomalous triple gauge boson vertex, K^0–$\overline{K}^0$ mixing, and the ρ-parameter, respectively. It is interesting that there is only one operator suppressed by a single power:

$$\mathcal{L}_5 = (LH)(LH). \quad (3)$$

After substituting the expectation value of the Higgs, the Lagrangian becomes

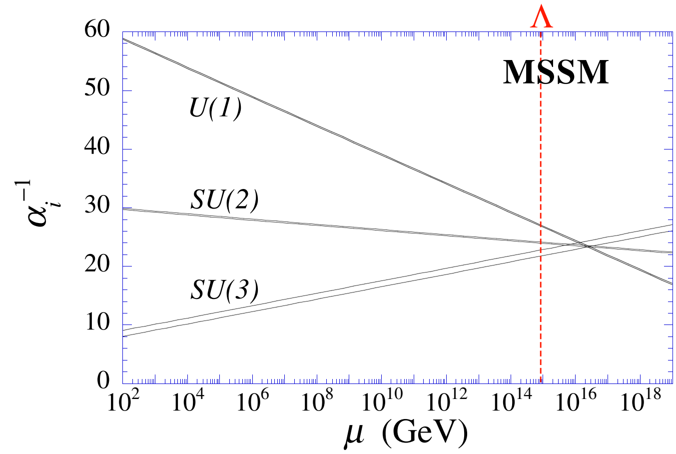

Fig. 1. Apparent unification of gauge coupling unification in the MSSM at 2×10^{16} GeV, compared to the suggested scale of new physics from the neutrino oscillation data

$$\mathcal{L} = \frac{1}{\Lambda}(LH)(LH) \to \frac{1}{\Lambda}(L\langle H\rangle)(L\langle H\rangle) = m_\nu \nu\nu, \qquad (4)$$

nothing but the neutrino mass.

Therefore the neutrino mass plays a very unique role. It is the lowest-order effect of physics at short distances. This is a very tiny effect. Any kinematical effects of the neutrino mass are suppressed by $(m_\nu/E_\nu)^2$, and for $m_\nu \sim 1$ eV which we now know is already too large and $E_\nu \sim 1$ GeV for typical accelerator-based neutrino experiments, it is as small as $(m_\nu/E_\nu)^2 \sim 10^{-18}$. At first sight, there is no hope to probe such a small number. However, any physicist knows that interferometry is a sensitive method to probe extremely tiny effects. For interferometry to work, we need a coherent source. Fortunately there are many coherent sources of neutrinos in Nature: the Sun, cosmic rays, reactors (not quite Nature), etc. We also need interference for an interferometer to work. Fortunately, there are large mixing angles that make the interference possible. We also need long baselines to enhance the tiny effects. Again fortunately there are many long baselines available, such as the size of the Sun, the size of the Earth, etc. Nature was very kind to provide all the necessary conditions for interferometry to us! Neutrino interferometry, a.k.a. neutrino oscillation, is a unique tool to study physics at very high energy scales.

At the currently accessible energy scale of about a hundred GeV in accelerators, the electromagnetic, weak, and strong forces have very different strengths. But their strengths become the same at 2×10^{16} GeV if the Standard Model is extended to become supersymmetric (Fig. 1). Given this, a natural candidate energy scale for new physics is $\Lambda \sim 10^{16}$ GeV, which suggests $m_\nu \sim \langle H\rangle^2/\Lambda \sim 0.003$ eV. On the other hand, the data suggest

$$m_\nu \sim (\Delta m^2_{\rm atm})^{1/2} \simeq 0.05 \text{ eV}, \qquad (5)$$

$$m_\nu \sim (\Delta m^2_{\rm solar})^{1/2} \simeq 0.008 \text{ eV}, \qquad (6)$$

quite close to this expectation. Therefore neutrino mass under our current studies may be probing physics at the scale of grand unification!

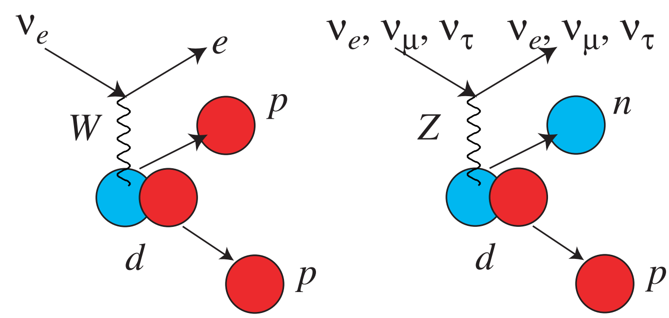

Fig. 2. Charged-current and neutral-current processes in SNO that measure ν_e flux and total active neutrino flux, respectively

3 Interpretation of data and seven questions

Since the previous EPS meeting in Budapest, 2001, there had been tremendous amount of new beautiful data. Here is a brief summary of what we have learned [1]:

- There is no doubt anymore that the atmospheric ν_μ's are lost. If you naively translate the latest χ^2 value from the SuperKamiokande experiment [2], the probability that the fluctuation can explain the data without oscillation is smaller than 4.2×10^{-26}. Of course, you wouldn't trust the χ^2 distribution down to this tail, but this number just demonstrates the level of clarity at which the deficit is now demonstrated by the data. Most likely, the lost ν_μ have converted to ν_τ instead of a sterile neutrino, at a confidence level higher than 99% [3].
- Solar ν_e's are converted to either ν_μ or ν_τ at more than 5 σ level [4].
- Reactor $\bar{\nu}_e$'s are lost at more than 99.95% confidence level [5].
- Only the so-called LMA (Large Mixing Angle) solution to the solar neutrino problem is left.
- All the data suggest strongly a tiny but non-vanishing neutrino mass, which is the first evidence for the *incompleteness of the Minimal Standard Model*, which withstood all experimental challenges for three decades.

3.1 Solar neutrino

I believe it is fair to say that the most remarkable achievement of the last two years is that

> *Solar Neutrino Problem is Finally Solved After 35 Years!*

There are two major pillars behind this statement.

The first one is the SNO result. We know that only ν_e's can be produced by the thermonuclear fusion in the Sun's core because even the core temperature of about 1 keV is nowhere close to the temperature needed to produce muons as heavy as 100 MeV, not to mention yet heavier taus. The beautiful measurements of the solar neutrino

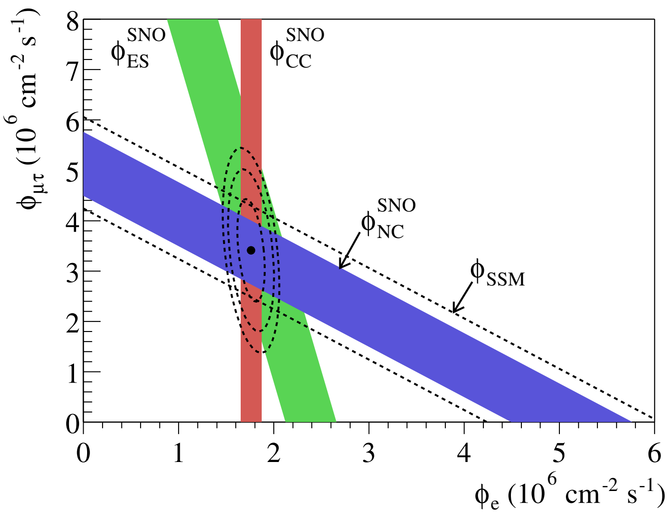

Fig. 3. Measurement of ν_e and $\nu_{\mu,\tau}$ fluxes from the Sun measured by SNO [6]

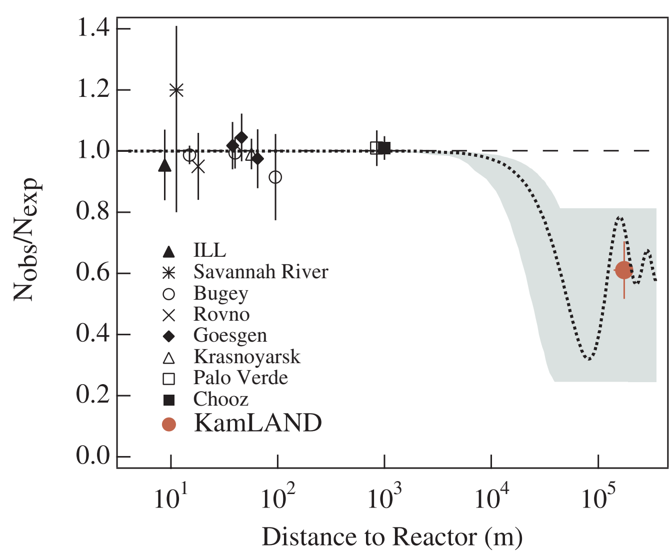

Fig. 4. Measurements of $\bar{\nu}_e$ survival probability by various reactor experiments at different distances, including the KamLAND result [8]

flux based on the charged-current and neutral-current reactions (Fig. 2)

$$\Phi_{CC} = (1.76 \pm 0.05 \pm 0.09) \times 10^6 \text{cm}^{-2}\,\text{sec}^{-1},$$
$$\Phi_{NC} = (5.09^{+0.44+0.46}_{-0.43-0.43}) \times 10^6 \text{cm}^{-2}\,\text{sec}^{-1}, \qquad (7)$$

differ by more than 5 σ [6].[2] It implies that there are *wrong* neutrinos, ν_μ and/or ν_τ, coming from the Sun. Somehow some of the ν_e's are converted to $\nu_{\mu,\tau}$ on their way from the Sun's core to the SNO detector (Fig. 3). It is the discovery of the *neutrino flavor conversion*!

With this amazing result alone, however, the neutrino flavor conversion could have been caused by a variety of mechanisms: neutrino decay, spin-resonant flip, a new exotic flavor-changing neutrino current, or violation of the equivalence principle.

The second pillar is the KamLAND result. It is the first terrestrial experiment relevant to the solar neutrino problem. Compared to the expected number of 86.8 ± 5.6 events calculated using the thermal power output of many contributing nuclear power plants, and the expected background of 0.95 ± 0.99 events, only 54 events were observed (Fig. 4). No oscillation hypothesis is excluded at the 99.95% confidence level.

Combination of SNO and KamLAND results, supplemented by other solar neutrino experiments, is extremely powerful, if CPT is assumed. To explain the solar neutrino problem, none of the above-mentioned alternatives to oscillation would work any more. Neutrino decay would give a wrong energy dependence in the solar neutrino survival probability. Spin-resonant flip relies on a strong solar magnetic field, while there is no such field between nuclear power plants and the KamLAND detector. A new exotic flavor-changing neutral current relies on a high solar matter density, while there is rather little matter between nuclear power plants and KamLAND. Violation of the equivalence principle employs a strong gravitational field around the Sun while there is no comparable field on Earth's surface. It leaves only the neutrino oscillation, and hence the neutrino masses and mixings, as the dominant mechanism for the neutrino flavor conversion.

Furthermore, there used to be disconnected regions of the parameter space over many decades of the mass-squared difference $\Delta m^2 \sim 10^{-11}$–10^{-3} eV^2 and the mixing angle $\sin^2 2\theta \sim 10^{-3}$–1 that could have been the solution (Fig. 5). The global fit to the solar neutrino rates used to give four such regions, called LMA, SMA (Small Mixing Angle), LOW (low Δm^2), or VAC (vacuum oscillation), with a possible bridge between the LOW and VAC regions sometimes called quasi-vacuum. Including the lack of time-dependence of the rates and the lack of distortion in the energy spectrum at SuperKamiokande practically eliminated SMA and most of VAC, leaving LMA and LOW. After SNO, the parameter range has converged down to LMA only. Correspondingly, the deficit observed at KamLAND would not occur for any other regions except for the LMA region. This is a remarkable focusing from many orders of magnitude of parameter range down to a factor of a few (Fig. 6).

Having witnessed the resolution of the decades-long puzzle, it is useful to recapitulate what the problem was and what the explanation is using the neutrino masses and mixings. The discussion below applies to the LMA region.

The solar neutrino spectrum had been calculated by John Bahcall since early 1960's and has been steadily refined to the BP2000 result [10] (Fig. 7). The problem is that we don't get enough of them (Fig. 8), suppressed to about a third for the higher energy components, the ^{7}Be and ^{8}B neutrinos. Within the two-flavor neutrino oscillation hypothesis, it is not possible to obtain a suppression by more than a factor of two for an energy-averaged oscillation. However, the matter effect [11] comes to the rescue.

[2] This has gotten even better with a later SNO result from its salt run [7].

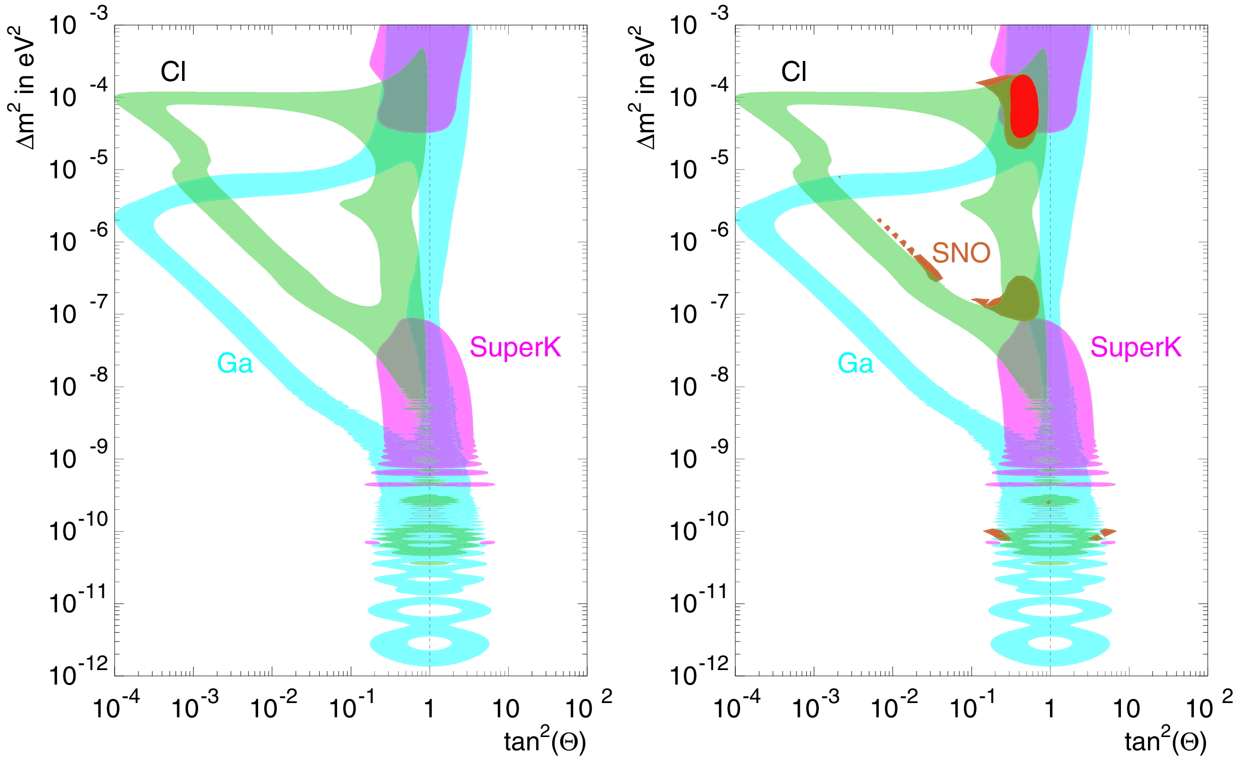

Fig. 5. The progress in solar neutrino in the year 2002. Before March and after April [9]

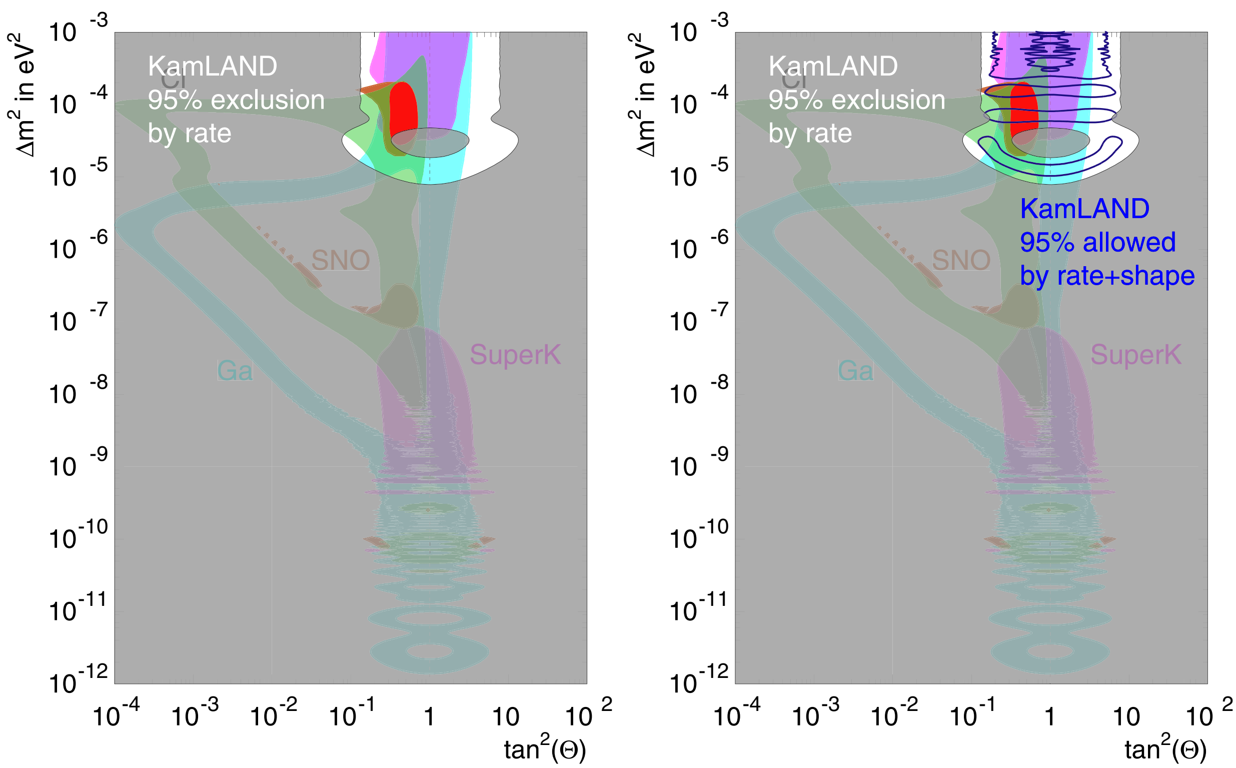

Fig. 6. The comparison of the solar neutrino data and the reactor anti-neutrino data after December [9]

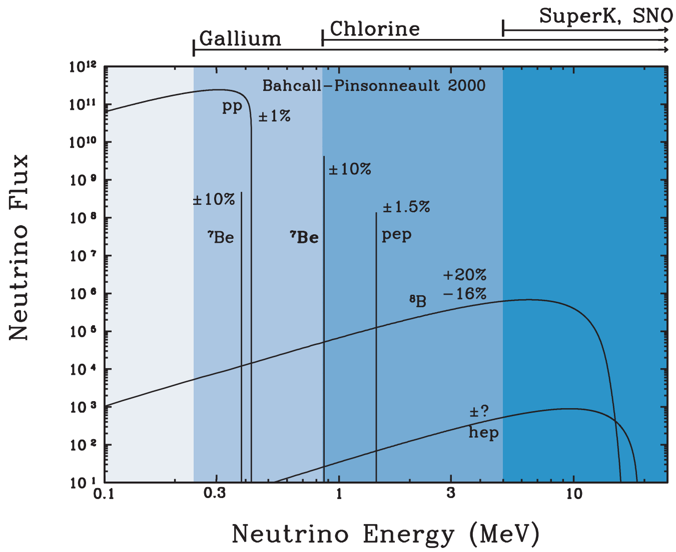

Fig. 7. The calculated solar neutrino flux BP2000

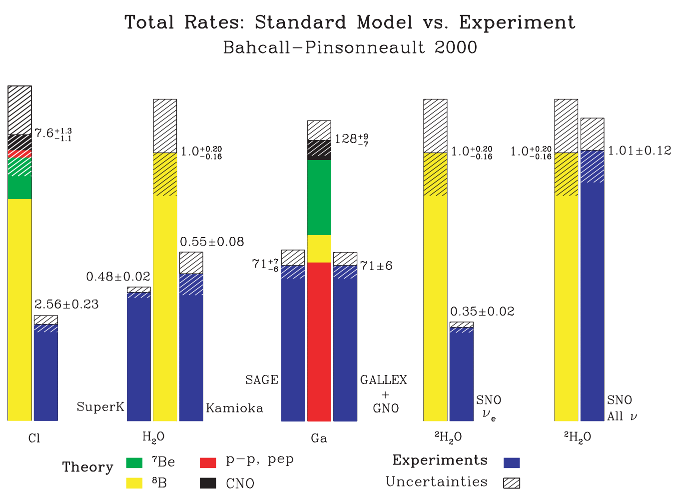

Fig. 8. The comparison of the BP2000 prediction and the experimental measurements

The charged-current interaction of ν_e in the presence of non-relativistic electrons gives a term in the Lagrangian

$$
\begin{aligned}
L_{\text{matter}} &= -\frac{G_F}{\sqrt{2}}(\bar{e}\gamma_\mu(1-\gamma_5)\nu_e)(\bar{\nu}_e\gamma^\mu(1-\gamma_5)e) \\
&= -\frac{G_F}{\sqrt{2}}(\bar{e}\gamma_\mu(1-\gamma_5)e)(\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e) \\
&= -\sqrt{2}G_F n_e(\bar{\nu}_e\gamma^0\nu_e),
\end{aligned}
\tag{8}
$$

where the second equality is just an algebraic identity, while the third equality follows because the only important component in the electron current operator is its time component which is nothing but the number density $\bar{e}\gamma_\mu(1-\gamma_5)e = (n_e, 0, 0, 0) + O(v/c)$. Because this term is proportional to the time component of the neutrino current, it can be interpreted as the potential energy term for the neutrino Hamiltonian. In the presence of neutrino masses and mixings, it reads

$$
H = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \sqrt{2}\,G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{9}
$$

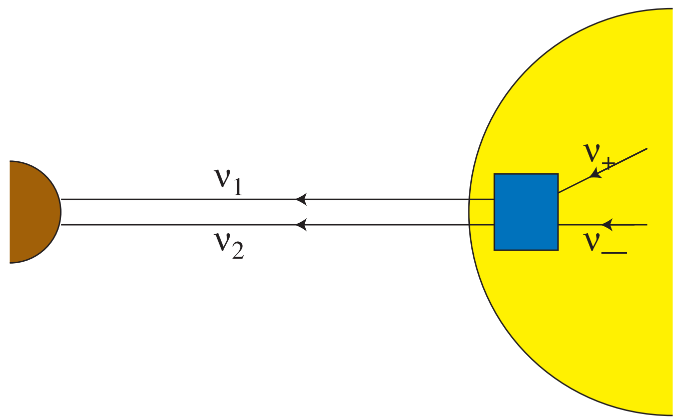

Fig. 9. The schematics of the matter effect as the ν_e produces in the core propagates through the Sun and exits as mass eigenstates if the process is adiabatic

Here, I dropped a piece proportional to the identity matrix that produces only a trivial overall phase in the wave functions. The last term makes the electron neutrino state have higher energy at the core. Because the electron number density n_e varies from the core to the surface of the Sun, the Hamiltonian is time-dependent.

The way it works for the LMA region is that the neutrinos evolve *adiabatically*. What it means is that the neutrinos are always in the "instantaneous" eigenstates of the Hamiltonian, even though a time-dependent Hamiltonian in general allows for coherent mixture of instantaneous eigenstates. See Figs. 9 and 10 to follow the discussion. When the neutrinos are produced in the core, they are strictly in the ν_e state, which can be decomposed using the instantaneous eigenstates $\nu_e = \nu_+ \sin\theta_m + \nu_- \cos\theta_m$. Here, ν_+ (ν_-) are the higher (lower) energy states at the core, while θ_m is the mixing angle used to diagonalize the Hamiltonian with the core density $n_e(0) \simeq 100 n_A/\text{cm}^3$ ($n_A = 6 \times 10^{23}$ is the Avogadro's number). The mixture of ν_+ and ν_- can be regarded completely incoherent because the averaging over the production point washes out the interference term that varies rapidly in the phase. Because the evolution of states is adiabatic, ν_+ (ν_-) always stays ν_+ (ν_-), even though their energy eigenvalues change as neutrinos traverse through the Sun, and hence they exit the Sun as an incoherent mixture of ν_+ with probability $\sin^2\theta_m$ and ν_- with probability $\cos^2\theta_m$. When the solar neutrinos are detected on Earth's surface using the charged-current reaction, we single out ν_e state out of ν_+ (ν_-), with the probabilities $\sin^2\theta$ ($\cos^2\theta$). Therefore the survival probability is given simply by (Fig. 11)

$$
P_{\text{surv}} = \cos^2\theta\cos^2\theta_m + \sin^2\theta\sin^2\theta_m. \tag{10}
$$

For the high-energy component such as the ^{8}B neutrinos, the $\Delta m^2/E$ term in (9) is negligible compared to the

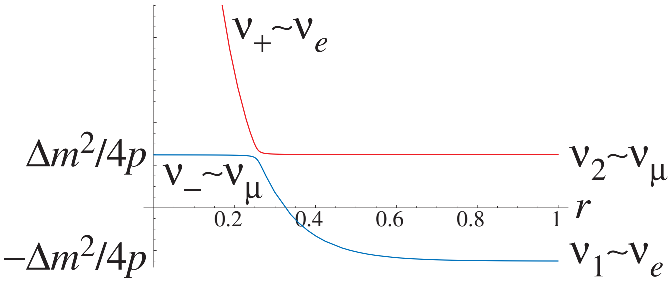

Fig. 10. The behavior of the instantaneous Hamiltonian eigenvalues as a function of the position inside the Sun for the LMA solution

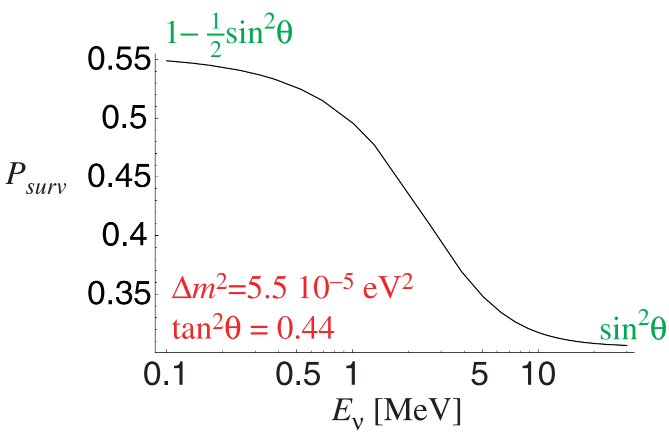

Fig. 11. The survival probability of the solar neutrinos for the LMA solution

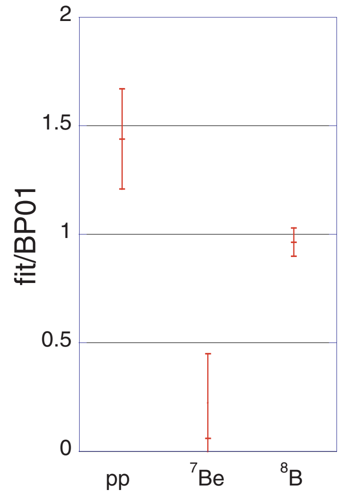

Fig. 12. The model independent determination of solar neutrino fluxes from data [12]

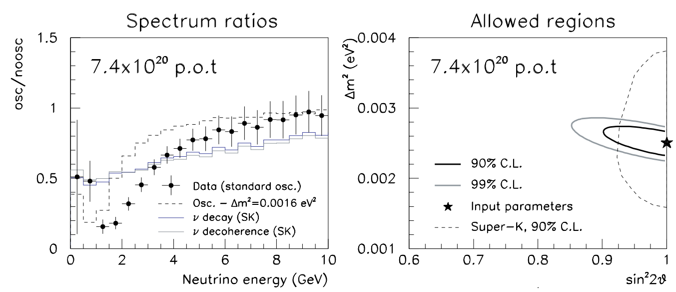

Fig. 13. The energy spectrum distortion in the MINOS data is expected to demonstrate the oscillation if Δm^2 is not too low. The expected accuracy in parameter measurements is also shown. Taken from [14]

matter term at the core, and the instantaneous Hamiltonian eigenstate is the same as the flavor eigenstate. It is important that the ν_e state is *higher*, and hence $\sin\theta_m = 1$, $\cos\theta_m = 0$. Then the survival probability is simply $P_{\rm surv} = \sin^2\theta$. This is how it can be as small as a third. On the other hand for the low-energy component such as the *pp* neutrinos, the mass term is enhanced as $\Delta m^2/E$, and the instantaneous Hamiltonian eigenstates are the same as the mass eigenstates, $\theta_m = \theta$. Then the survival probability is $P_{\rm surv} = \cos^4\theta + \sin^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$, which is the same result as the energy-averaged vacuum oscillation. It is clearly larger than a half.

Having understood the relevant neutrino properties, it is finally time to do what Ray Davis and John Bahcall wanted to do forty years ago: to probe physics of the Sun using neutrinos [12]. One can fit all solar neutrino data together with KamLAND to *measure* three major components of the solar neutrino spectrum, *pp*, ^{7}Be, and ^{8}B (Fig. 12). The current errors in neutrino oscillation parameters are still significant for this purpose, but nonetheless the solar luminosity inferred from the extracted neutrino fluxes gives

$$\frac{L_\odot(\nu)}{L_\odot(\text{light})} = 1.4^{+0.2}_{-0.3}, \tag{11}$$

confirming the thermonuclear fusion process indeed generates the amount of light we see.[3]

The evidence for the neutrino oscillation in the atmospheric and solar neutrinos appears extremely strong. There are still loose ends we'd like to tie up, however. First, the energy dependence of the survival probability crucial in the explanation of the solar neutrino data is not fully demonstrated. It calls for new solar neutrino experiments capable of energy spectrum measurement down to the *pp* and ^{7}Be regions. They would allow for unambiguous verification for the matter effect. The survival probability at the *pp* energy region would directly determine θ_{12}, which helps with the interpretation of future long-baseline oscillation experiment aiming at the CP violation, and also with the neutrinoless double-beta decay experiments as I'll discuss later.

The second loose end is that nobody has actually seen "oscillation," namely the periodic variation of the survival probability with dips and peaks. The MINOS experiment in Minnesota uses the NuMI beam from Fermilab to see

[3] If the density perturbation in the Sun is unexpected large, however, it may severely limit our ability to calculate the survival probability and makes this comparison meaningless [13].

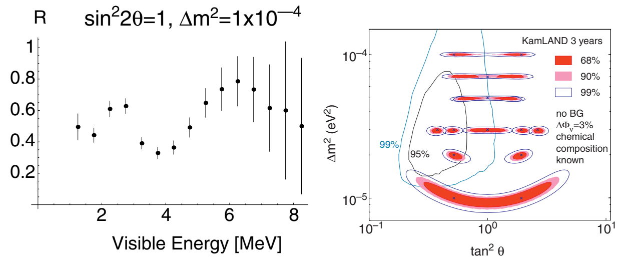

Fig. 14. The energy spectrum distortion in the future KamLAND data is expected to demonstrate the oscillation if Δm^2 is not too large. The expected accuracy in parameter measurements is also shown. Taken from [15]

the oscillation dip relevant for the atmospheric neutrino oscillation (Fig. 13). The location of the dip determines Δm_{23}^2, while the depth the mixing angle θ_{23}. The accuracy will be improved further using the neutrino beam from J-PARC to SuperKamiokande [16] and/or an off-axis NuMI experiment [17]. A longer exposure of KamLAND will demonstrate the oscillation dip if Δm^2 is not too high, giving a precise determination of Δm_{12}^2, but θ_{12} measurement will likely suffer from small statistics. Ironically, there is no way to see the "oscillation" in solar neutrinos because they are incoherent mixture of two mass eigenstates for the LMA region.

Third, the evidence for ν_τ "appearance" in atmospheric neutrinos is still not strong enough, only about 99% CL from SuperKamiokande. The OPERA [18] and ICARUS [19] experiments will detect τ appearance directly using the CNGS beam from CERN under construction.

3.2 Interpretation without LSND

How do we put together all the existing data within the three-generation framework? One major question is the unconfirmed evidence for neutrino oscillation from the LSND experiment. I first discuss the case where this evidence will be definitively refuted in the future.

The standard parameterization of the MNS neutrino mixing matrix for three generations is

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}. \tag{12}$$

To a very good approximation, the first factor is relevant to the solar neutrinos, while the last one to the atmospheric neutrinos. On the other hand, the middle factor is still somewhat poorly constrained, mostly by the previous generation of reactor neutrino experiments CHOOZ [20] and Palo Verde [21].

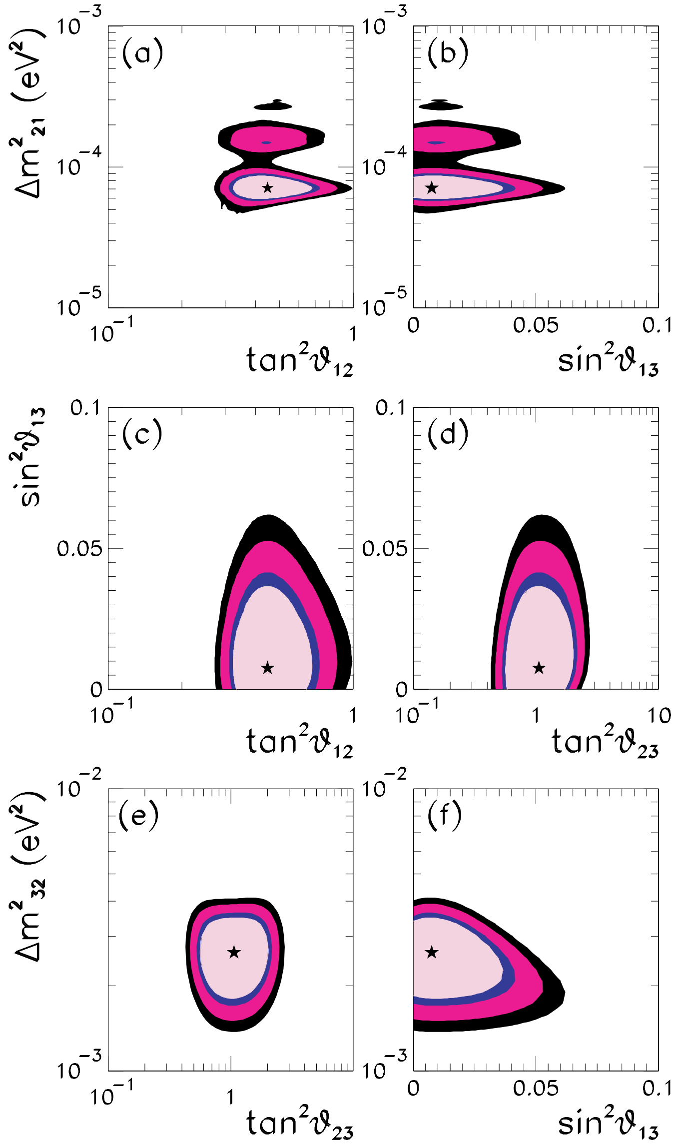

Fig. 15. The global fit of the solar, reactor, atmospheric neutrino data in the three-generation framework, taken from [22]

The most important point to emphasize is that the current solar, reactor, and atmospheric neutrino data as well as the K2K accelerator-based data can easily be accommodated within the three generation framework. The basic results from the global fit are as follows (Fig. 15). $\sin^2 2\theta_{23}$ is near maximal, and $\Delta m_{23}^2 \simeq 2 \times 10^{-3}$ eV^2. On the other hand, $\sin^2 2\theta_{12}$ is large but not maximal, with $\Delta m_{12}^2 \simeq 7 \times 10^{-5}$ eV^2. The last mixing angle $\sin^2 2\theta_{13} = 4|U_{e3}|^2 \lesssim 0.05$ is smallish, constrained by CHOOZ and Palo Verde. Because of the small $\sin^2 2\theta_{13}$, the solar (reactor) neutrino oscillation almost decouples from the atmospheric neutrino oscillation. Therefore, the "piecewise" two-flavor oscillation analysis is a good approximation to the solar (reactor) and atmospheric neutrino results.

Despite this consistent picture of neutrino oscillation physics, we are still left with many big questions. There are *Seven Questions*:

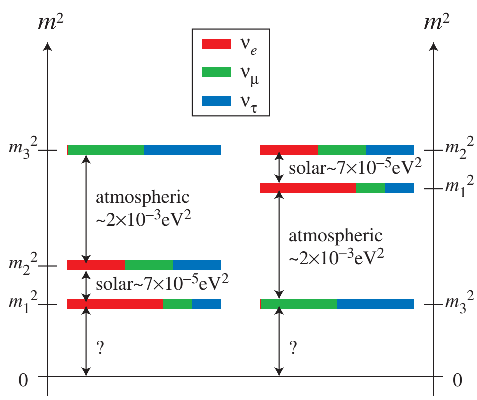

Fig. 16. Our current best knowledge on neutrino masses and mixings. The spectrum may be either "normal" (*left*) or "inverted" (*right*)

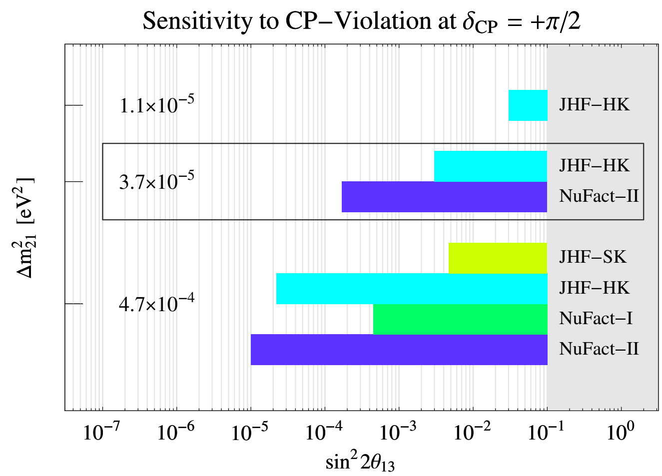

Fig. 17. The sensitivity to CP violation for $\delta_{CP} = +\pi/2$ at the 90% confidence level, plotted as function of $\sin^2 2\theta_{13}$ [24]

- Dirac or Majorana?
- Absolute mass scale?
- How small is θ_{13}?
- CP Violation?
- Mass hierarchy?
- Verify oscillation?
- LSND? Sterile neutrino(s)? CPT violation?

The first question is in some sense an embarrassment. We don't know if the anti-neutrinos are the same particles as the neutrinos. I feel like I can't answer an innocent question by my seven-year old daughter. Because we are using the "neutrino interferometry" to study neutrino masses and mixings, we can study only the relative phase, namely the mass-square difference Δm^2, but not the absolute phase, namely the masses themselves. The mixing angle θ_{13} is only poorly constrained at this moment. It appears smallish, but that is all we know. How small is it? We are very eager to know if there is CP violation in the neutrino sector. This is especially of interest if we would like to link the cosmic baryon asymmetry to the neutrinos. Thanks to the beautiful oscillation data, we know two different mass-squared differences Δm^2_{12} and Δm^2_{23}, and therefore there are $2! = 2$ ways to order them (Fig. 16). We do not know if the neutrino masses have the "normal hierarchy," where two light states have the smaller mass splitting, or the "inverted hierarchy," where two heavy states are nearly degenerate. As I mentioned earlier, none of us has "seen" the oscillation of neutrinos yet. It would be important to see it. Finally, if the LSND evidence turns out to be correct, it will turn things upside down.

Within the framework of three neutrinos, let me discuss the following three questions first: θ_{13}, CP violation, and mass hierarchy.

Now that the LMA solution is confirmed, it is a dream case for neutrino oscillation physics. This is because Δm^2_{12} is within the reach of long-baseline neutrino oscillation experiments. That would make CP violation in neutrino oscillation a possible target. The CP violation can be seen as a difference between the neutrino oscillation and anti-neutrino oscillation, such as [23]

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) = -16 s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin\delta \sin\left(\frac{\Delta m^2_{12}}{4E} L\right) \sin\left(\frac{\Delta m^2_{13}}{4E} L\right) \sin\left(\frac{\Delta m^2_{23}}{4E} L\right). \tag{13}$$

Because this expression is a product of many factors, it can be sizable only if all of the factors are large enough. Fortunately, some of the factors had been known to be large enough, such as the atmospheric mixing angle s_{23} and the associated mass-squared difference $\Delta m^2_{23} \simeq \Delta m^2_{13}$. In addition, it is observably large only if the factors associated with the solar (reactor) neutrino oscillation, Δm^2_{12} and s_{12}, as well as the unknown angle s_{13} are all large enough. The LMA actually gives the highest possible end for Δm^2_{12}, and the angle is also large even though not maximal. The only remaining unknown then is θ_{13} and the CP-violating phase δ itself. Therefore, there are high hopes that the CP violation is probable using a so-called neutrino superbeam, the high-intensity version of the conventional horn-based neutrino beam, or eventually the muon-storage ring neutrino factory [26] (Fig. 17). Nature has been kind to us again. Moreover, the presence of the matter effect in the Earth, if the baseline is long enough, would allow us to discriminate between the normal and inverted hierarchies.

It is clear that the numerical value of θ_{13} decides the future of this field. Its value determines the required facility, parameters, baseline, and energy of the experiment. There are two paths discussed in the community to measure its value: long-baseline accelerator-based experiment, and medium baseline (a few kilometers) reactor anti-neutrino experiment. Both directions are under active study (Fig. 18) .

3.3 Interpretation with LSND

What about the LSND evidence of neutrino oscillation? The LSND experiment used the stopped positive muon

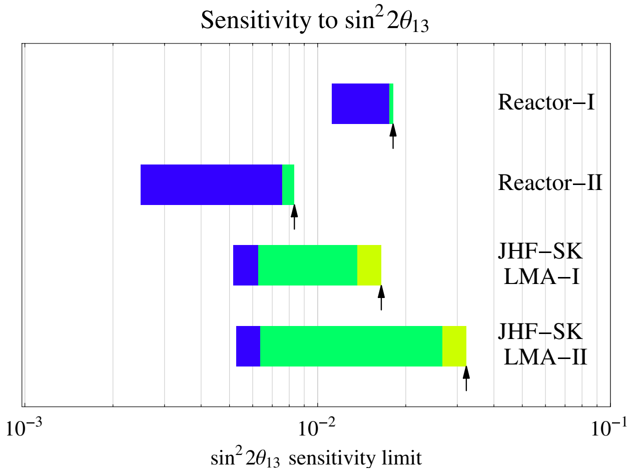

Fig. 18. The sensitivity to $\sin^2 2\theta_{13}$ for two different options for reactor experiment, J-PARC to SuperK at LMA-I ($\Delta m^2_{12} = 7 \cdot 10^{-5}\,\mathrm{eV}^2$), and at LMA-II ($\Delta m^2_{12} = 1.4 \cdot 10^{-4}\,\mathrm{eV}^2$) at the 90% confidence level, after successively switching on systematics (*dark/blue*), correlations (*medium gray/green*), and degeneracies (*light gray/yellow*) [24]

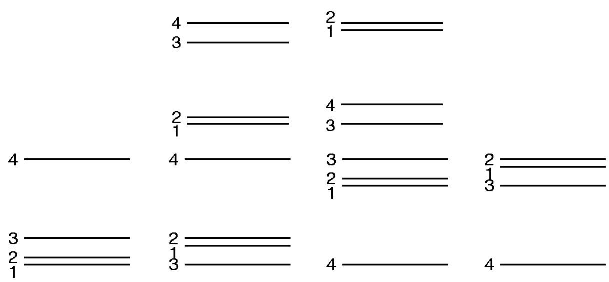

Fig. 19. The $2+2$ and $3+1$ spectra

and its decay $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$. It looked for the appearance of $\bar{\nu}_e$ from the oscillation of $\bar{\nu}_\mu$ in a liquid scintillator via the classic inverse beta decay $\bar{\nu}_e p \to e^+ n$ followed by the delayed capture of the neutron. It reported the positive signal of the appearance [27],

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = (0.264 \pm 0.067 \pm 0.045)\%. \qquad (14)$$

If I naively combine the statistical and systematic errors in quadrature, it would be a 3.3σ signal. The reported significance is even higher than that. If interpreted as an oscillation signal, it suggests a relatively high Δm^2 of about 0.2–2 eV^2. The Mini-BooNE experiment at Fermilab is designed and has started physics run to settle this reported evidence for good [28].

The problem is that it does not fit into the three-neutrino framework I have just discussed. We now have two well-established values for Δm^2, one for the atmospheric $\Delta m^2 \simeq 2 \times 10^{-3}$ eV^2, and the other for the solar (reactor) $\Delta m^2 \simeq 7 \times 10^{-5}$ eV^2 neutrino oscillation. With three neutrino states, there are only two linearly independent Δm^2. But together with LSND, we need three. They cannot be consistently accommodated with three neutrinos. Therefore the LSND evidence is usually discussed with new exotic neutrino species. The invisible Z-width from LEP does not allow any new neutrino species in the Z-decay, and hence the new species should not couple to the Z-bosons; hence *sterile* neutrinos. Now with three different mass-squared differences, there are $3! = 6$ ways to order them (Fig. 19). Four of them are called $3+1$ spectrum because there is a close triplet widely separated from a singlet by Δm^2_{LSND}. Two remaining ones are called $2+2$ spectrum because there are two close doublets, one for $\Delta m^2_{\mathrm{solar}}$ and the other for $\Delta m^2_{\mathrm{atm}}$, separated by Δm^2_{LSND}.

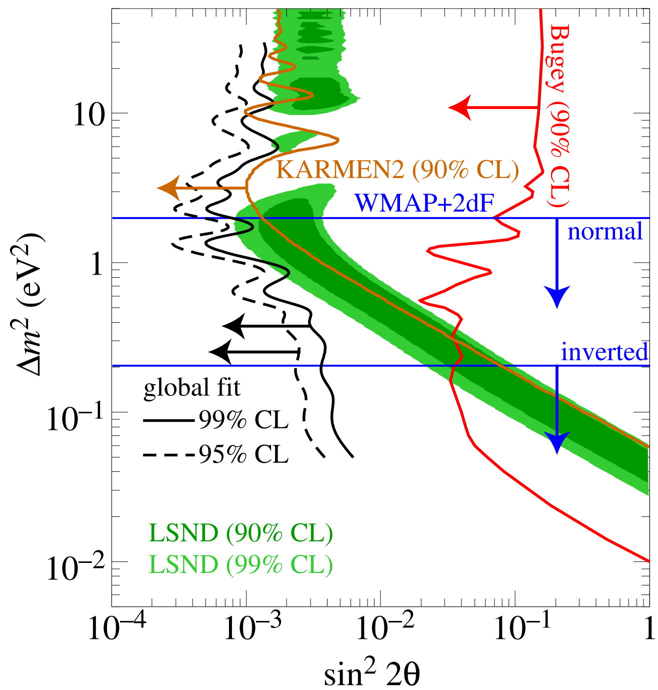

Fig. 20. The tension between LSND and global fit to other data from [31], combined with the cosmological constraint from [33], as presented in [34]

Past global fits used to prefer the $2+2$ spectrum. However, it is now seriously disfavored after the SNO result. In this spectrum, the sterile state must be distributed somehow between two doublets. However, the atmospheric neutrino data from SuperKamiokande suggests that the $\nu_\mu \to \nu_s$ oscillation is disfavored at more than 99% CL, and the atmospheric doublet cannot contain much of the sterile state. On the other hand, the agreement of the BP2000 solar neutrino flux calculation and the SNO result of the neutral current flux means that the solar neutrinos cannot convert much to the sterile state either, which is the new constraint. Therefore, neither doublet can have much of the sterile component, and this spectrum is possible only at the 1.6×10^{-6} level [31].

On the other hand, the $3+1$ spectrum has been constrained by a combination of short-baseline oscillation experiments. In this spectrum, the LSND mixing angle is an effective parameter given by a product of mixing angles

$$\sin^2 2\theta_{LSND} = 4|U_{e4}|^2 |U_{\mu 4}|^2,$$

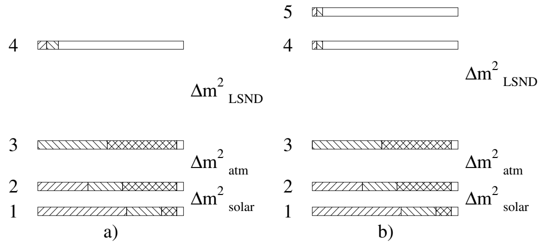

Fig. 21. 3 + 1 and 3 + 2 spectra, from [30]

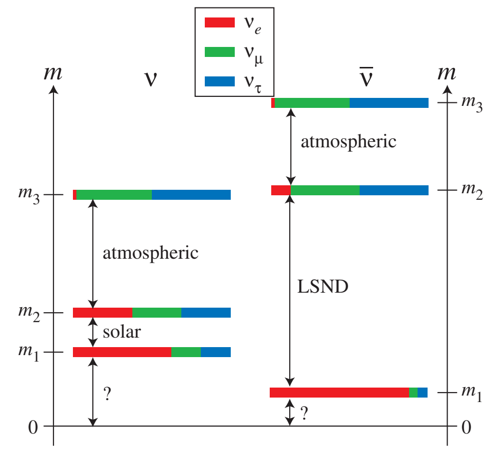

Fig. 22. The original suggestion for CPT-violating mass spectrum to reconcile LSND with other data [36]

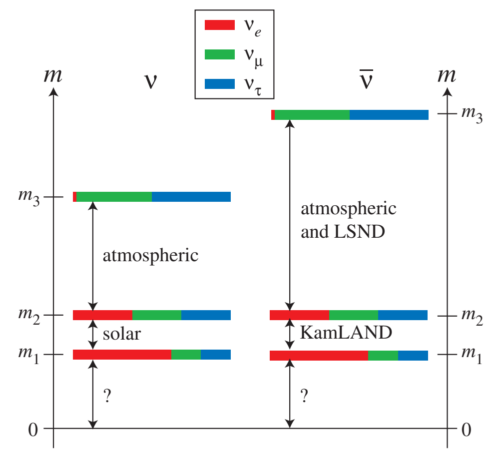

Fig. 23. The revised suggestion for CPT-violating mass spectrum to reconcile LSND with other data after KamLAND [38]

where the mass eigenstate 4 refers to the widely separated singlet. Even though the LSND data allows $\sin^2 2\theta_{LSND}$ as small as 10^{-3}, the individual mixing angles U_{e4} and $U_{\mu 4}$ cannot be that small. On the other hand, $U_{\mu 4}$ is constrained to be small by the lack of ν_μ disappearance in CDHSW as well as the near maximal depletion in the up/down ratio of atmospheric neutrinos at SuperKamiokande. Correspondingly, U_{e4} is constrained to be small because of the lack of $\bar{\nu}_e$ disappearance in the Bugey reactor neutrino experiment. This scenario is also disfavored, with the probability of 5.6×10^{-3} [31]. There are only small regions consistent with both the LSND (99%CL) and other experiments combined (99%CL) (Fig. 20). Neither one looks good.

A prominent theorist once said, "Sterile neutrino is like cockroach. If there is one, there must be more" [29]. It is natural to ask then if more sterile neutrinos would help. After all, there could well be one for each generation. By adding the second sterile neutrino, obviously there are more parameters, reducing the tension just because of the degrees of freedom in the χ^2 analysis. But a recent study [30] shows that there is also a small improvement in the χ^2_{min} itself.

Sterile neutrino has always been at odds with cosmology. The successful agreement between the observed abundance of light elements, deuterium, ^{3}He, ^{4}He, and ^{7}Li with the predictions of the Big-Bang Nucleosynthesis theory would be spoiled if there are too many neutrino species because they would have altered the expansion rate of the universe back then. Even though the precise constraint is always subject to discussions, one additional species appears quite difficult to accommodate, and adding two or more is regarded excluded (see, *e.g.*, [34]). Moreover, the presence of additional neutrino species with Δm^2 of eV range implies that the total amount of energy density in neutrinos at present is quite high, $\Omega_\nu h^2 = \sum_i m_{\nu_i}/97$ eV. The WMAP collaboration reported an upper limit of $\sum_i m_{\nu i} < 0.7$ eV, by combining their normalization of the CMBR anisotropy together with the 2dF galaxy survey and the Lyman α data [32]. A naive comparison seems to suggest that the regions marginally left by the combination of the LSND and other short-baseline experiments violate this limit. A more careful study showed, however, that a larger number of neutrino species actually relaxes this limit and the LSND regions are not completely excluded [33], while some of the spectra, such as 1 + 3 inverted one, is in conflict [34] (Fig. 20). One needs to resort to some mechanism that prevents the sterile states to be produced from neutrino oscillation in early universe, such as a large lepton asymmetry [35].

One suggestion to remedy this situation is the possibility of CPT violation [36]. It relies on a simple observation: the LSND evidence is based on anti-neutrinos, while the solar evidence is on neutrinos. Therefore, if neutrinos and anti-neutrinos have different mass spectra (Fig. 22), it allows for *four* independent mass-squared differences, and can accommodate the solar, atmospheric, and LSND evidences with three neutrinos alone. One can therefore assign $\Delta m^2_{\rm solar}$ and $\Delta m^2_{\rm atm}$ to neutrinos, and $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm LSND}$ to anti-neutrinos. This framework used to be actually the best fit to the data.

However, KamLAND changed the situation completely. It observed *anti-neutrino* deficit consistent with the solar *neutrino* deficit, and hence one mass-squared difference in neutrinos and another in anti-neutrinos must be practically the same. It can be argued that it is actually the best limit on CPT violation, surpassing that from the long-time champion neutral kaon system [37]. A revised suggestion was to assign $\Delta m^2_{\rm solar}$ and $\Delta m^2_{\rm LSND}$

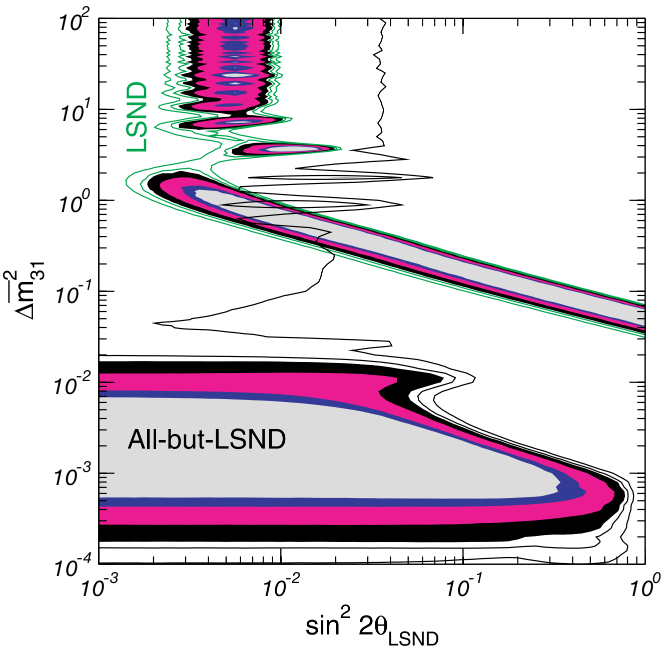

Fig. 24. The CPT violation does not reconcile the LSND evidence with other data either, taken from [39]

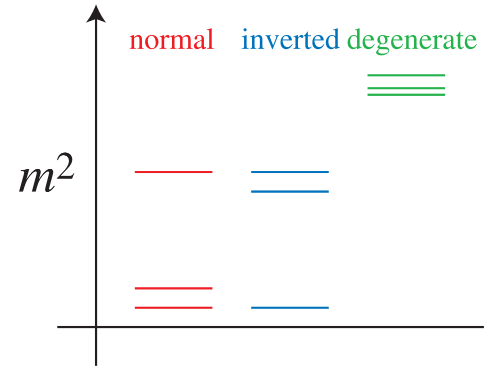

Fig. 25. Three possible mass spectra of neutrinos

to anti-neutrinos instead [38] (Fig. 23). It was argued viable because the atmospheric neutrino signal is about three-quarters neutrinos and one-quarter anti-neutrinos, and hence the SuperKamiokande result constrains anti-neutrinos weakly. However the reduced sensitivity still constrains Δm^2 for anti-neutrinos and the global fit is not good for this framework (Fig. 24). MINOS can study atmospheric neutrinos and in principle separate neutrinos from anti-neutrinos thanks to the magnetized iron plates, and settle this issue.

If neither sterile neutrino nor CPT violation is the answer, what else could be? One suggested possibility is that the LSND evidence is not neutrino oscillation, but other exotic physics. For instance, it may have detected an anomalous muon decay $\mu^+ \to e^+ \bar{\nu}_\mu \bar{\nu}_e$ so that $\bar{\nu}_e$ is already there in the decay product [40]. Note that this suggested decay mode not only violates the lepton family number but also the overall lepton number as well. The KARMEN experiment disfavors the anomalous decay, putting a limit of $BR(\mu^+ \to e^+ \bar{\nu}_\mu \bar{\nu}_e) < 0.009$ (90% CL) [41], while the explanation of the LSND evidence requires $BR = 0.019$–0.040, in a (weak) tension. This framework does not lead to any observable consequences at Mini-BooNE. On the other hand, it predicts a subtle effect on the Michel parameter in the muon decay, $\rho \simeq 0.7485$, to be compared to the Standard Model prediction $\rho = 0.75$ because of the $V - A$ structure of the charged-current weak interaction. The current experimental accuracy $\rho = 0.7518 \pm 0.0026$ does not provide a clear discrimination. The TWIST experiment aims at the measurement of the Michel parameter down to a few times 10^{-4} level, and is expected to settle this question [42].

In any case, a definitive test of the LSND signal at Mini-BooNE is eagerly awaited, and if confirmed, it is clear that we have to rethink everything from scratch.

3.4 Nature of Neutrino Mass

The first two of the Seven Questions, Dirac or Majorana, and the absolute mass scales, are tough but very important ones. Having learned that the neutrinos have mass, the Minimal Standard Model is incomplete. We have to extend it somehow to incorporate the neutrino masses and mixings. But how exactly do we extend it? We have to abandon something in the Minimal Standard Model. One possibility is to abandon the minimality of the particle content, namely to introduce new unobserved light degrees of freedom (right-handed neutrinos). The other possibility is to abandon the lepton-number conservation, namely to abandon fundamental distinction between neutrinos and anti-neutrinos, and hence matter and anti-matter. The first case would give us Dirac neutrinos, while the latter Majorana neutrinos. Without knowing which is the case, we do not know how to extend the Standard Model to incorporate the new discovery into the theory.

Among theorists, there has been a bias to prefer Majorana neutrinos. That is because of the so-called seesaw mechanism [43] that naturally explains the smallness (but finiteness) of the neutrino masses. If we simply add right-handed neutrinos to the Standard Model, the structure is completely analogous to other fermions, and therefore we would naively expect neutrino masses similar to charged leptons or quarks. Clearly, this expectation is violated. But then one realizes that the right-handed neutrinos are somewhat special: they do not carry any quantum numbers under the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. No color, singlet under the weak interaction, and no charge. Therefore, adding mass to the right-handed neutrinos would not violate any gauge symmetry unlike the case for charged leptons and quarks. Then the mass is not tied to the scale of the Higgs needed to break the gauge symmetry and make the fermion masses possible. In fact, the masses of right-handed neutrinos can be arbitrarily high, even up to the Planck scale $M_{Pl} = 2 \times 10^{18}$ GeV. Then the mass matrix of one-generation left- and right-handed neutrinos is given by

$$(\nu_L, \nu_R) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \qquad (15)$$

where m_D is expected to be of the same order of magnitude as the charged-lepton or quark masses, while M is extremely large, $M \gg m_D$. Because the determinant of the mass matrix is $-m_D^2$ while the trace M, one eigenvalue is basically M, while the other one is extremely suppressed, $m_\nu = -m_D^2/M$. This way, one can understand why the neutrinos are special and have such tiny but finite masses as a consequence of physics at an extremely high-energy scale, reminiscent of Weinberg's discussion I presented at the beginning of this talk. And this mechanism implies that the neutrinos are Majorana particles.

The only known practical approach to discriminate Majorana vs Dirac neutrinos is to look for neutrinoless double beta decay or $0\nu\beta\beta$, a nuclear transition that involves $nn \to ppe^-e^-$ without accompanying neutrino emission. The matrix element of this process is proportional to the so-called effective neutrino mass $\langle m_{ee} \rangle = \sum_i m_{\nu_i} U_{ei}^2$. One important point is that it depends on the overall mass scale of neutrinos, not just the mass-squared differences as in neutrino oscillation. Another point is that it is sensitive to the phase of U_{ei}^2, which are in general *complex* parameters. The current limit is $|\langle m_{ee} \rangle| \lesssim 0.2$–$1$ eV, where the actual number depends on the nuclear matrix element that is not known very well. Typically, we can ignore the term $m_e U_{e3}^2$ in the sum because of the small-ish U_{e3}, and the expression simplifies to

$$\langle m_{ee} \rangle \approx m_1 \cos^2\theta_{12} + e^{i\phi} m_2 \sin^2\theta_{12}. \qquad (16)$$

Here, $e^{i\phi}$ is an unknown phase that does not appear in neutrino oscillation. If $e^{i\phi} = -1$, two terms can cancel to some extent.

Fortunately, there is a new piece of information from SNO. The angle θ_{12} is not maximal,

$$\cos^2\theta_{12} - \sin^2\theta_{12} = \cos^2 2\theta_{12} > 0.07\ (1\sigma), \qquad (17)$$

and the two terms cannot cancel exactly if $m_1 \simeq m_2$.[4]

The degree of the possible cancellation depends on what kind of spectrum neutrinos have. There are three different possibilities, normal, inverted, or degenerate (Fig. 25).

In the degenerate spectrum, all neutrinos are at $m \gtrsim 0.1$ eV with splittings much smaller than masses themselves. It is still possible even after the WMAP limit $m_\nu < 0.23$ eV. In this case, one can imagine that KATRIN [45], a new improved experiment on the tritium end point, or improved cosmological data, such as Planck satellite [46] and Sloan Digital Sky Survey [47], would confirm this large neutrino masses. If so, we will predict

$$|\langle m_{ee} \rangle| = m \cos^2 2\theta_{12} > 0.07m \qquad (18)$$

even allowing for the maximal possible cancellation $e^{i\phi} = -1$. A negative result from the $0\nu\beta\beta$ experiments that exclude this range would eliminate the possibility of three Majorana neutrinos.

[4] See an improved limit using more recent data in [44].

The inverted spectrum may be confirmed by the future long-baseline neutrino oscillation experiments, if θ_{13} is not too small. In this case, $m_3 \sim 0$, while $m_1 \approx m_2 \approx (\Delta m^2_{\rm atm})^{1/2} \approx 0.05$ eV. Therefore,

$$|\langle m_{ee} \rangle| = m \cos^2 2\theta_{12} > 0.0035 \text{ eV}. \qquad (19)$$

If the $0\nu\beta\beta$ experiments exclude this range, they would eliminate the possibility of three Majorana neutrinos.

In the normal spectrum, however, $m_1 \approx m_2 \approx 0$ and $m_3 \approx (\Delta m^2_{\rm atm})^{1/2} \approx 0.05$ eV, and $|\langle m_{ee} \rangle|$ can in principle vanish exactly even if neutrinos are Majorana. In this case, there is no guarantee that we can settle this question. We can only hope that future data would bring some clarity either way.

It is interesting to recall the WMAP limit once again, $m_\nu < 0.23$ eV each at 95% CL. A simple inequality shows that

$$|\langle m_{ee} \rangle| = \left| \sum_i m_{\nu_i} U_{ei}^2 \right| < \sum_i m_{\nu_i} |U_{ei}|^2 < 0.23 \text{ eV}. \qquad (20)$$

The reported evidence for $0\nu\beta\beta$ by (a subset of) the Heidelberg–Moscow experiment suggests $|\langle m_{ee} \rangle| = 0.11$–$0.56$ eV [48], while a reinterpretation using different nuclear matrix elements suggests $|\langle m_{ee} \rangle| = 0.4$–$1.3$ eV [49]. At least a portion of the suggested range is in conflict with the WMAP limit [34]. It is interesting to watch how these numbers will evolve in the future.

4 Models of flavor

I have mentioned earlier that the order of magnitude of neutrino masses came out more or less consistent with the theoretical suggestions based on the seesaw mechanism and the grand-unified theories. In fact, it is interesting to see how well theorists have been doing in predicting the neutrino properties.

Back around 1990, a typical neutrino theorist said:

– The solution to the solar neutrino problem *must* be the small-angle MSW solution because it is beautiful.
– Natural scale for Δm^2_{23} is 10–100 eV^2 because it is cosmologically interesting.
– The angle θ_{23} must be similar to $V_{cb} \simeq 0.04$.
– The atmospheric neutrino anomaly must go away because its explanation in terms of neutrino oscillation requires a large angle.

As you must have noticed, all these predictions turned out to be wrong. Clearly, theorists have an excellent track record in neutrino physics. `;-)`

What are the surprises? Most of us had been prejudiced that mixing angles are small and masses are hierarchical because of the quarks and charged leptons. On the other hand, now that the LMA solution has been estab-

Table 1. Models based on different flavor quantum number assignments, taken from [51]. After SNO and KamLAND, only the first two remain viable

Model	parameters	d_{23}	$\Delta m^2_{12}/\lvert\Delta m^2_{23}\rvert$	U_{e3}	$\tan^2\theta_{12}$	$\tan^2\theta_{23}$
A	$\epsilon = 1$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$
SA	$\epsilon = \lambda$	$O(1)$	$O(d_{23}^2)$	$O(\lambda)$	$O(\lambda^2/d_{23}^2)$	$O(1)$
$\mathtt{H_{II}}$	$\epsilon = \lambda^2$	$O(\lambda^2)$	$O(\lambda^4)$	$O(\lambda^2)$	$O(1)$	$O(1)$
$\mathtt{H_{I}}$	$\epsilon = \lambda^2$	0	$O(\lambda^6)$	$O(\lambda^2)$	$O(1)$	$O(1)$
IH (LA)	$\epsilon = \eta = \lambda$	$O(\lambda^4)$	$O(\lambda^2)$	$O(\lambda^2)$	$1 + O(\lambda^2)$	$O(1)$
IH (LOW)	$\epsilon = \eta = \lambda^2$	$O(\lambda^8)$	$O(\lambda^4)$	$O(\lambda^4)$	$1 + O(\lambda^4)$	$O(1)$

lished, we know that the mixing angles are mostly large,

$$U_{MNS} = \begin{pmatrix} \text{big} & \text{big} & \text{medium?} \\ \text{big} & \text{big} & \text{big} \\ \text{big} & \text{big} & \text{big} \end{pmatrix}, \qquad (21)$$

while the two mass splittings are not very different,

$$\frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} = 0.01\text{–}0.2\ . \qquad (22)$$

The ratio is not much smaller than unity especially if you take its square root. In particular, the atmospheric mixing angle θ_{23} appears *maximal*. A pressing question then is if there is any new symmetry or structure behind this newly discovered pattern of the neutrino masses and mixings.

Obviously, this is a part of the bigger question of flavor [50]. What distinguishes different generations? They share exactly the same gauge quantum numbers, yet they look different. Their masses are hierarchical and they mix little. It suggests a need for some ordered structure. It is against a "common sense" in quantum mechanics. States that have the same set of quantum numbers are expected to have similar energy levels and mix a lot under tiny perturbations. Somehow, quarks and charged leptons do not behave that way. What it means is probably that there exists a hidden *flavor quantum number* that distinguishes different generations. Noether told us that a new quantum number means a new symmetry, a flavor symmetry. This symmetry must allow the top quark Yukawa coupling because it is $O(1)$. However, all the other Yukawa couplings are presumably forbidden by the flavor symmetry. Only after the flavor symmetry breaks by a small order parameter, small Yukawa couplings become allowed and are generated. Different generations mix little because they have different quantum numbers, and can mix only by the symmetry breaking effects. Masses are very different because the states (three generation) have different quantum numbers.

The question then is what the symmetry is. To find the underlying symmetry from the data, we would like to repeat what Heisenberg (isospin) or Gell-Mann–Okubo (flavor $SU(3)$) did.

The masses and mixings of the quarks, charged leptons, and neutrinos suggest various possible flavor symmetries. For example, Table 1 shows a list of different models based on different flavor quantum number assignments consistent with data. The list was made in October

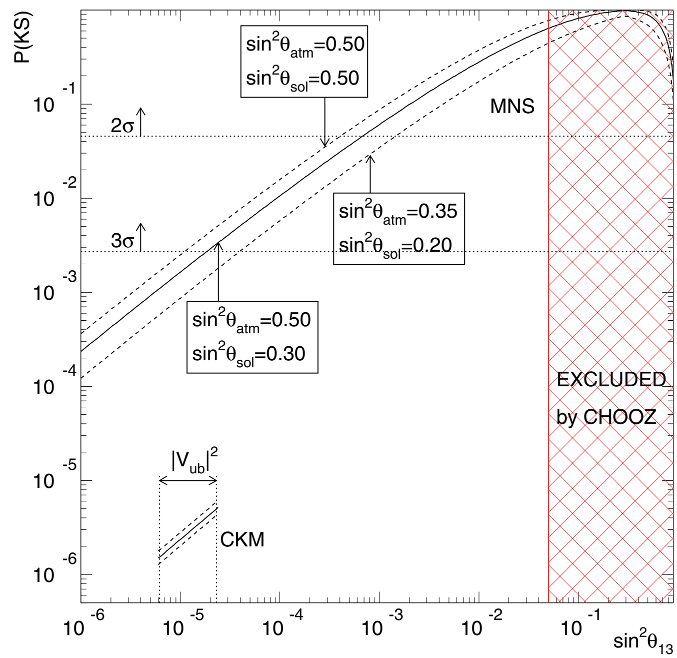

Fig. 26. $P(KS)$ for the MNS matrix as a function of $\sin^2\theta_{13} \equiv |U_{e3}|^2$ [53]

2002, while the SNO and KamLAND data have already excluded all but the first two rows. Data have narrowed down possible flavor symmetries.

Let me discuss the first one briefly because it is *mine* [52]. It is based on the idea that the lack of symmetry explains the observed neutrino masses and mixings. Suppose there is no fundamental distinction among three neutrinos. Then their mass matrix is expected to consist of entries of comparable size. It should look like a random three-by-three matrix of $O(1)$ coefficients. Then one can show that the MNS matrix is distributed according to the group-theoretical measure (Haar measure). It turns out that the maximal angle is rather natural, because the distribution is flat in $\sin^2\theta_{23}$, and hence peaked at maximal in $\sin^2 2\theta_{23}$. Three angles, θ_{23}, θ_{12}, and θ_{13}, would be three random draws from the same distribution peaked at maximal $\sin^2 2\theta = 1$, and it is quite natural that one of them (θ_{13}) comes out smallish. Indeed the Kolmogorov–Smirnov test suggests that the observed pattern is consistent with this idea at 64% probability [53] (Fig. 26). We called this

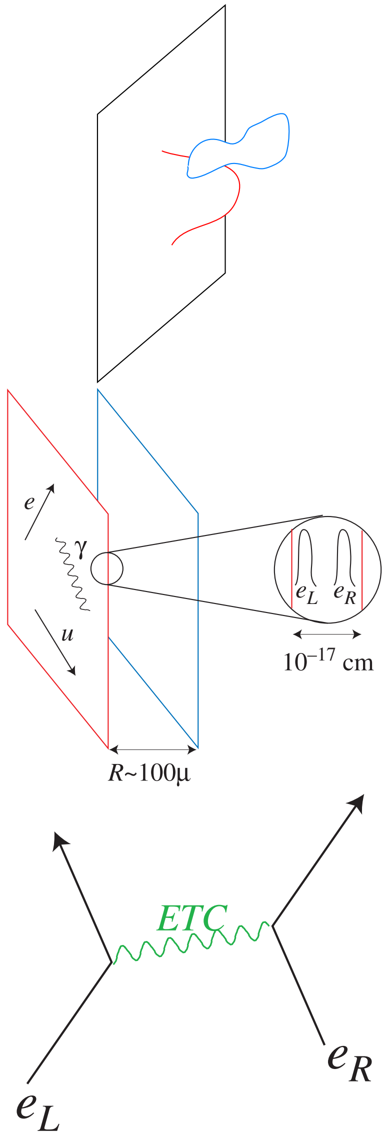

Fig. 27. Different views on the origin of flavor symmetry depending on the outcome of the TeV-scale physics. *Top*: string origin in Supersymmetric models. *Center*: physical dislocation of different generations within a fat brane in models with hidden dimensions. *Bottom*: exchange of new massive gauge bosons at 100 TeV scale in technicolor models

hypothesis, namely that three neutrinos share the same flavor quantum number, the "anarchy."

The key prediction of the anarchy is that θ_{13} should not be too small, either. $\sin^2 2\theta_{13}$ should be larger than 0.01 at 99% CL.

The main lesson from this discussion is that there are several critical measurements that will further narrow down the possible flavor quantum number assignments.

– How maximal is θ_{23}? If an improvement measurement shows $\sin^2 2\theta_{23} = 1.00 \pm 0.01$, it will call for a new symmetry that forces the angle to be maximal. On the other hand, if it is large but not quite maximal, an anarchy-like hypothesis will be preferred.

$$\begin{pmatrix} \tilde{s}_R \\ \tilde{s}_R \\ \tilde{s}_R \\ \tilde{\nu}_\mu \\ \tilde{\mu} \end{pmatrix} \leftrightarrow \begin{pmatrix} \tilde{b}_R \\ \tilde{b}_R \\ \tilde{b}_R \\ \tilde{\nu}_\tau \\ \tilde{\tau} \end{pmatrix}$$

Fig. 28. The large $\nu_\mu \to \nu_\tau$ mixing suggests a large mixing of the whole $SU(5)$ multiplets and also of their superpartners

– How small is θ_{13}? If $\sin^2 2\theta_{13} < 0.01$, it strongly suggests different flavor quantum numbers of ν_e from ν_μ or ν_τ.
– Is the mass spectrum normal or inverted? Most flavor symmetries predict the normal hierarchy, but there are exceptions.
– Is there CP violation? It will provide a plausibility test of the leptogenesis.

Once we identify a flavor symmetry, the next obvious question would be the dynamics behind it. In the case of the flavor $SU(3)$, the dynamics turned out to be the QCD. The possibilities will be completely different depending on what the true theory of electroweak symmetry breaking is (Fig. 27). If it is supersymmetry, the flavor symmetry may be an apparently anomalous $U(1)$ gauge symmetry made anomaly-free by the Green–Schwarz mechanism in the string theory [54]. If it is large extra dimensions, the difference in flavor quantum numbers may be due to the different physical locations of three generations inside the "fat brane" [55]. If it is technicolor, the flavor symmetry arises from a sequential breaking of extended technicolor gauge symmetry [56]. The question of flavor is not stand-alone. We need information from the energy frontier.

We need information from the quark sector, too. Here is one specific example we should pursue [58]. We'd like to know if quarks and leptons have a common origin of flavor. We know that the ν_μ and ν_τ mix *a lot*, maybe even maximally. Suppose you make it grand-unified. s_R lives in the same multiplet as ν_μ, and b_R with ν_τ. You'd expect a large mixing between s_R and b_R, too (Fig. 28). But mixing among right-handed quarks completely drops out from the CKM phenomenology because there is no right-handed charged current (as far as we know). It looks like we can't probe this question. On the other hand, if there is supersymmetry, a large mixing between $\tilde{s}_R$ and $\tilde{b}_R$ is physical, and can induce $O(1)$ effects in $b \to s$ transitions through loop diagrams (Fig. 29, top and center). Especially in leptogenesis that relies on CP-violation in the neutrino sector [57], we expect CP-violation in $\tilde{s}_R$–$\tilde{b}_R$ mixing that may show up in B-physics.

For example, we may see CP-violation in B_s mixing that can be studied in $B_s \to J/\psi + \phi$. The rates in $B_d \to X_s \ell^+ \ell^-$ may differ from the Standard Model and CP-violation may be seen. CP-violation in $B_d \to \phi + K_S$ may be different from that in $J/\psi + K_S$ within all the other constraints, such as $b \to s\gamma$ (Fig. 29, bottom).

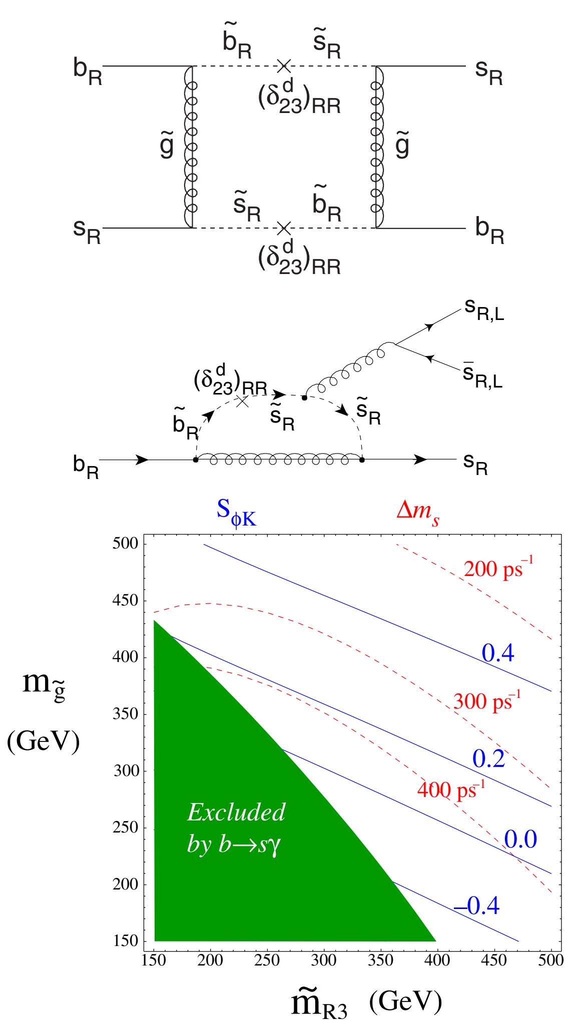

Fig. 29. The impact of large $\tilde{s}_R$–$\tilde{b}_R$ mixing on B-physics. *Top*: possible contribution to the B_s mixing. *Center*: possible contribution to the $B_d \to \phi K_S$ decay. *Bottom*: $S_{\phi K}$ in *solid lines*, Δm_s in *dotted lines*, and the constraint from $b \to s\gamma$ in *shaded region* [59]

5 Conclusions

We are going through a revolutionary stage in neutrino physics. There has been enormous progress in data, and in fact, the decades-long solar neutrino problem is now solved. There are, however, still some loose ends, and many forthcoming experiments will address them. If the LSND evidence is set aside, the three-generation oscillation framework works very well. The situation with the LSND evidence is still unclear. None of the suggested possibilities, namely an addition of sterile neutrino(s), CPT violation, or lepton-flavor violating anomalous muon decay, seem to work very well with the existing experimental constraints, but the jury is still out. Cosmological constraints are beginning to be interesting.

The next key parameter is θ_{13}. Two approaches are being pursued, one using the conventional accelerator-based neutrino beam, the other using reactor anti-neutrinos. Its value will decide the future of the field. If it is not too small, the nature is exceedingly generous once more; we may see CP violation in neutrino oscillation using a neutrino superbeam. In any case, neutrino physics continues to be exciting.

One point I'd like to emphasize is that the neutrino physics does not stand alone. For us to figure out the origin of the neutrino masses and mixings, we need to combine the data from neutrino physics with the information from the energy frontier as well as the quark sector.

Now, do you think you need theorists?

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